Analysis Channelizers with Even and Odd Indexed Bin Centers

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Abstract-The standard M-path analysis channelizer center frequencies coincide with the M sampled data frequencies of the M-point DFT, the frequencies with integer number of cycles per length of M-samples. These are the M multiples of fs/M, the frequencies that alias to DC when their sinusoids are down sampled M-to-1. The spacing between center frequencies is also fs/M as is the output sample rate when maximally decimated. There is a channelizer variation that have its center frequencies offset by the half channel spacing. These center frequencies are located midway between the DFT frequencies and contain (2M+1)/2 cycles per interval per length of M-samples. In this channelizer the index 0 is not the center frequency of the baseband channel but rather the crossover frequency of the adjacent bins centered at ±0.5 cycles per interval. The filters have the same bandwidth and have the same sample rate of the DFT bin centered channelizer. Changes to the standard channelizer to obtain the offset channelizer require a complex heterodyne of the input series or a complex heterodyne of the filter coefficients. In this paper we present interesting and useful modifications to the channelizer structure that avoids the complex heterodyne when converting between the channelizer options. By avoiding the complex multiplies at the input sample rate, the modified channelizers have a reduced signal processing workload.

Keywords – offset bin-centered channelizers, even and odd center frequency channelizers, modified polyphase filter bank, non maximally decimated even and odd bin centered channelizers, non-maximally decimated; offset center frequencies.

I. INTRODUCTION

The M-path analysis channelizer center frequencies coincide with the M sampled data frequencies of the M-point DFT, the frequencies with integer number of samples per length of M-samples. These are the M multiples of fs/M, the sinusoid frequencies that alias to DC when they are down sampled M-to-1. The spacing between center frequencies is fs/M as is the output sample rate when maximally decimated. The first channelizer variation we examine is shown in Figure In that FFT, there is the same symmetry of the spectral points about index 0 as there is about index 8 (or M/2). We often use MATLAB's *fftshift* command to interchange index 0 and index M/2 for display purposes. This exchange preserves the spectral symmetries of an even length FFT but it is not preserved for an odd length FFT. Let us consider a DFT for an odd number of points, say 15 for example. Such a DFT can be implemented by a Good-Thomas (GT) algorithm or by a conventional mixed radix Cooley-Tukey (CT) algorithm. An advantage of using the GT transform is there are no twiddle factors in the algorithm and the arithmetic is performed with

1. Here we present the spectra of two channelizers with equally spaced center frequencies, say 2 MHz, but with different center frequency locations. In the upper subplot, the center frequencies reside on half the even integer frequencies $\Delta f \cdot (2k)/2$ while in the lower subplot, the center frequency reside on half the odd integer frequencies $\Delta f \cdot (2k+1)/2$. The filters have the same shape, bandwidth, and sample rate in their respective implementations.

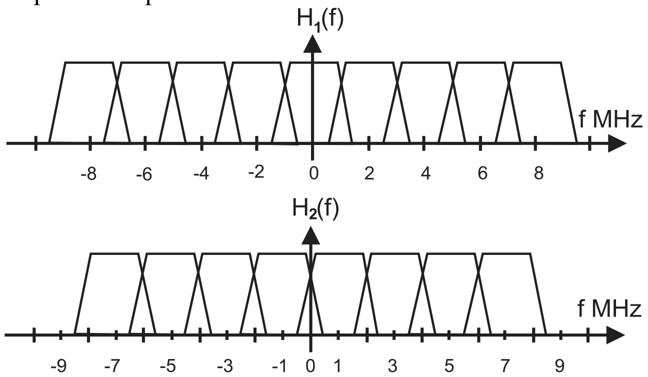

Figure 1. Spectra of channelizers with even and with odd indexed center frequencies with same channel shape, bandwidth, and frequency spacing. Upper subplot centers match DFT center frequencies centered on half the even integers. Lower subplot centers are offset by half their spacing centered on half the odd integers.

The standard response to the problem that a signal and a filter do not reside at the same center frequency is to move one of them: the signal to the filter (by the Armstrong heterodyne) or the filter to the signal (using the Equivalency theorem). These two options are shown in Figure 2. In both cases, a complex heterodyne at the input rate is required to perform the spectral alignment. While the frequency shift of the input signal or of the filter frequency response solves the offset problem, it does so at some cost. Rather than shift the input signal's spectrum or the filter's spectrum half a bin width, we can consider a much larger, spectral shift, but a shift less expensive to implement. We start by examining the FFT that implements the DFT. Here we discuss the FFT even though the channelizer uses the IFFT because we more easily visualize frequency bins when we see the FFT. Many FFTs are implemented by the radix-2 Cooley-Tukey algorithm which is a transform for an even number of points, say 16 for example.

real arithmetic and requires fewer arithmetic operations. As a side note, a 16 point CT FFT requires 36 real multiplies while a 15 point GT FFT requires 10 real multiplies.

Figure 3 shows the root locations of $Z^{15} - 1$ which corresponds to the center frequencies of a 15 point DFT. On the left subplot, the zero frequency location of an unaltered input sequence is indicated on the circle. This of course coincides with index 0 of the 15 point DFT. On the right subplot, the zero frequency location of the input sequence following a heterodyne to the half sample rate by alternating signs is indicated at the half sample rate on the circle. The DC term is

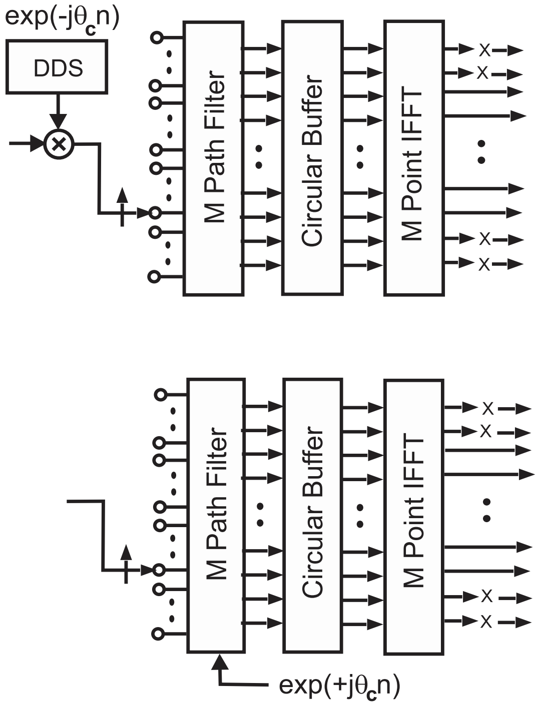

Figure 2. Aligning spectra of input signal with spectral responses of filter bank by complex heterodyne of input signal in upper subplot or by complex heterodyne of filter coefficient weights in lower subplot.

seen to reside midway between indices 7 and 8 of the 15 point DFT. This means that the indices 7 and 8 correspond to the two frequencies below and above DC by half the channel spacing. In this process, we do not have to a apply complex heterodyne to the input series or to the filter weights to access the half bandwidth offset frequency channelizer responses. The interaction of the odd length DFT and the alternating sign input heterodyne place the offset input frequency centers in the DFT bin centers. We simply have to relabel the bin indices to the offset center frequencies of the half sample rate rotated input spectrum. The mapping from bin index k to center frequency index f_k is shown in (1). For this example, if fs=150 MHz and M=15, frequency f_8 is shown in (2) to be +5 MHz.

$$f_k = \left(k\frac{f_S}{M} - \frac{f_S}{2} \right) \tag{1}$$

$$\begin{aligned} f_8 &= \left(8\frac{150}{15} - \frac{150}{2} \right) \text{MHz} \\ &= (8\times 10 - 75) = 5 \text{ MHz} \end{aligned} \tag{2}$$

II. Input Heterodyne to Filter Weights

In the previous section we learned the benefit of selecting an M-path channelizer with M selected to be odd integer. We used the fact that while DC resided on an FFT index, the half sample rate resided midway between a pair of FFT indices. The input heterodyne of DC to the half sample rate placed bin centers offset from DC by half the channel spacing. We still have to access alternate input samples to perform sign reversals. While we have avoided the complex rotation we are still

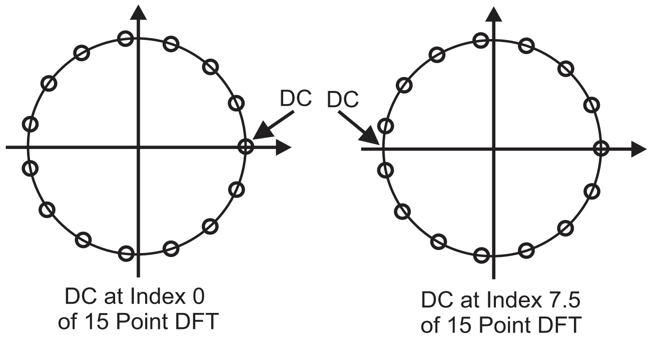

Figure 3. Two Unit Circles with Roots of ($Z^{15} - 1$), the Frequencies Corresponding to a 15 point DFT. The Left Subplot Indicates the Location of DC or Zero Frequency of an Unaltered Input Sequence Presented to the DFT. The Right Subplot Indicates the Location of DC or Zero Frequency Heterodyned to the Half Sample Rate by an Alternating Sign Heterodyne of the Input Sequence.

accessing input samples at the high input sample rate. We wonder if we can use the odd length FFT with the embedded offset at the half sample rate but avoid the heterodyne of the signal to the half sample rate. The question suggests that we are not quite finished with this *thinking outside the box* example. We now examine how the alternating sign input data interacts with the filter coefficients. Figure 10.4 shows the input data index and the data signs for two successive inputs of 15 new input samples to 15 point polyphase filter operating in its maximally decimated form. Note the sign reversals of the corresponding sample positions in the two new input vectors. These sign reversals cause the path outputs to have the desired sign reversals of the input heterodyne. We could use a state machine with embedded sign reversals in the polyphase filter coefficients to obtain the same sign flipping behavior seen in Figure 4, but a different option quickly presents itself.

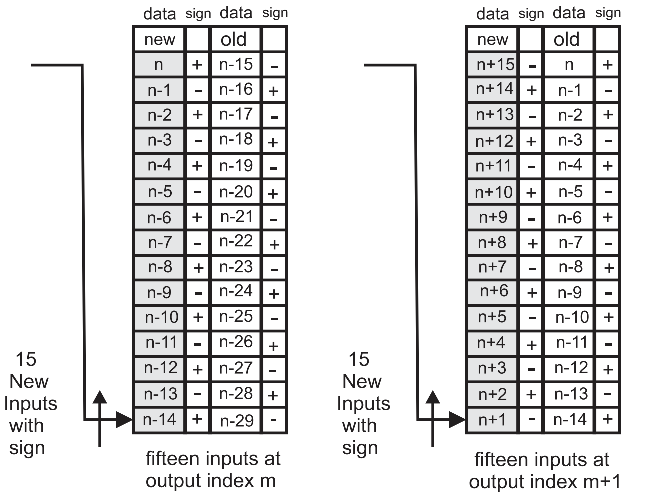

Figure 4. Polyphase Filter Input Sample Indices and Sign of Input Heterodyne for Two Successive 15-Point Data Samples in 15-Path Polyphase Filter.

Figure 10.5 shows the input data index and the alternating data signs for two successive inputs of 10 new input samples to 15 point polyphase filter operating in its non-maximally

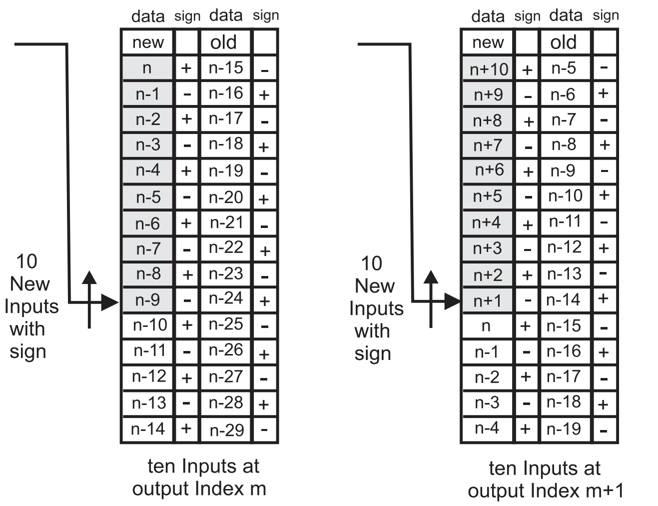

Figure 5. Polyphase Filter Input Sample Indices and Sign of Input Heterodyne for Two Successive 10-Point Data Sample Sequences in 15-Path Polyphase Filter. Note there are no Sign Reversals of the Two New Input Vectors

decimated 10-to-1 down sampling form. We note that there are no sign changes in corresponding positions of successive 10-sample input vectors in the non-maximally decimated version of the 15 path filter. This is because the length of the successive input vectors is, 10 which is a multiple of the 2 sample period of the sign changes of the input heterodyne. Here it comes! Because the signs don't change on successive inputs, we can associate the signs with the filter weights. That is, rather than heterodyne the input samples to the half sample rate at the input sample rate, we heterodyne the filter weights as an off-line operation. This is an interesting version of the

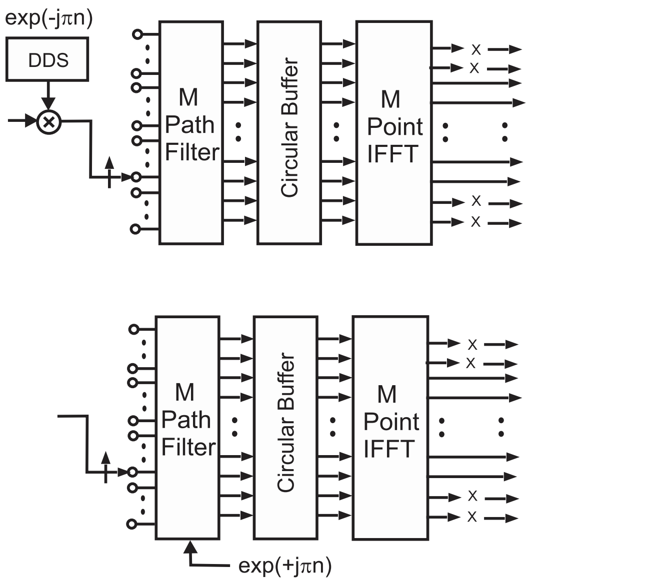

Figure 6. Aligning Spectra of Input Signal with Spectral Responses of Odd Length, Non-Maximally Decimated Filter Bank by Alternating Sign Heterodyne of Input Signal in Upper Subplot or by Alternating Sign Heterodyne of Filter Coefficient Weights in Lower Subplot.

equivalency theorem embedded in the polyphase filter. Figure 6 shows the application of the equivalency theorem to the non-maximally decimated filter bank formed by an odd length polyphase filter. Interestingly there is no on-line signal processing required to obtain the odd-indexed filter centers in this version of the M-path filter. Figure 7 shows the input and output spectrum formed by the 15-path polyphase filter with alternating sign heterodyne embedded in filter weights. This is a very nice result.

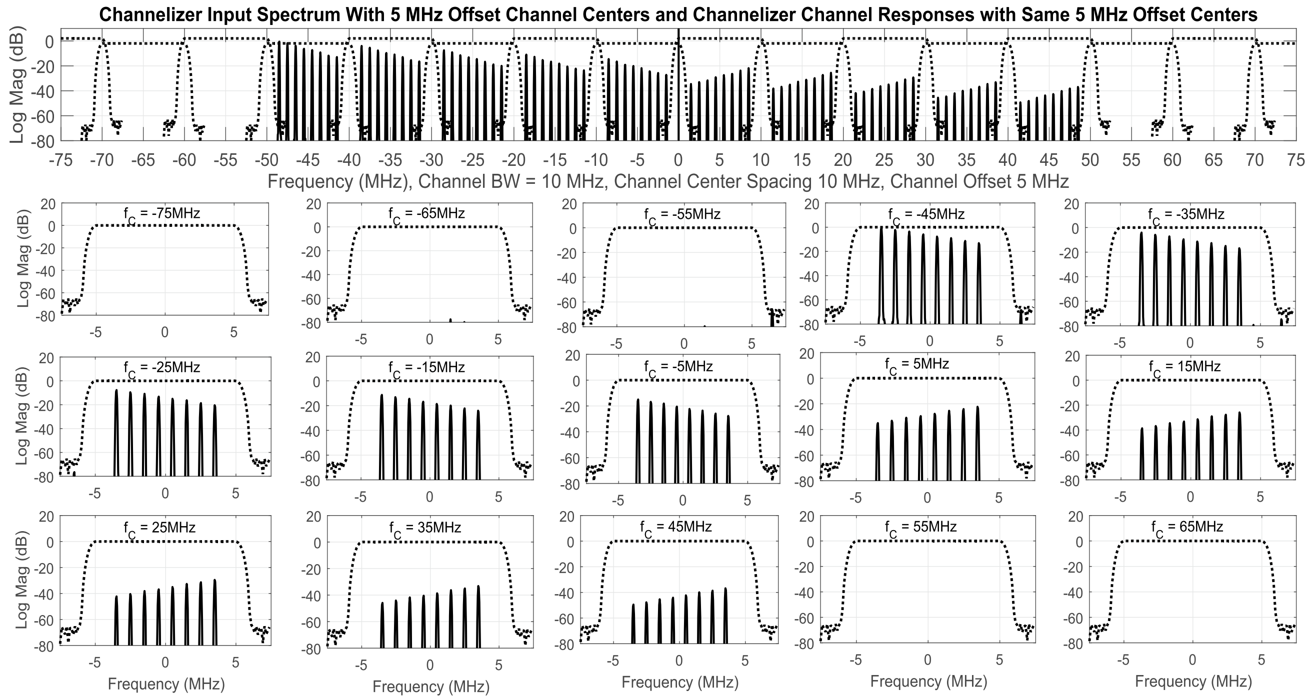

Figure 10.7. Spectra of Input Signal and Channel Centers of 15-Path Polyphase Channelizer Performing 10-to-1 Down Sampling with Alignment of Channelizer Spectra with Half-Channel Bandwidth Offset Performed by Embedding Alternating Sign Heterodyne in Filter Weights. Lower 15 Subplots Show Spectra Obtained at Each Baseband Channel Output Port.

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Our last comments on this equivalency theorem application follows. If you have need of an even length transform you would lose the half sample rate being located midway between DFT frequency indices. We can still use the spectral location between DFT indices at the quarter sample rate. As an example, Figure 10.8 shows DC at index 0 of an 18 point DFT without the heterodyne and midway between indices 4 and 5 of the 18 point DFT as a result of an input heterodyne by exp(j n π/2). To be able to embed the phase shifts in the polyphase filter the down sample rate P must be a multiple of 4 to keep the phase changes stationary in the filter on successive inputs of length P. We demonstrated successful operation of this modified process with an 18-path filter and 18-point FFT performing 12-to-1 down sampling. There is, of course, a re-indexing required to locate the shifted frequency centers at the offset DFT output indices.

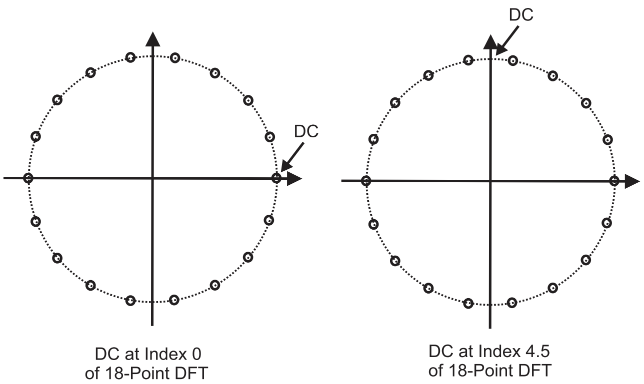

Figure 8. Two Unit Circles with Roots of (Z^{18} – 1), the Frequencies Corresponding to an 18 point DFT. The Left Subplot Indicates the Location of DC or Zero Frequency of an Unaltered Input Sequence Presented to the DFT. The Right Subplot Indicates the Location of DC or Zero Frequency Heterodyned to the Quarter Sample Rate by exp(j n π/2) Heterodyne of the Input Sequence.

III. CLOSING COMMENTS

We have described a clever way to implement an M-channel analysis channelizer with frequency bin centers offset from DC by half their channel spacing. This bin location variation is traditionally referred to as odd indexed bin centers. The reason designs use the odd indexed bin centers is we can form a symmetric allocation of channels with an even number of bin centers. When we have the even indexed bin centers, the symmetric channel assignment have an odd number of channels with one channel centered at DC which may or may not be occupied. Many OFDM based systems avoid centering a channel at DC due to the DC bin corruption by various DC intrusion sources. These sources include analog mixers self-mixing components, ADC truncation quantization of input samples, and 2's complement bias due to truncation arithmetic. The traditional response to aligning the bin centers of an analysis channelizer with the offset bin centers of a multichannel odd indexed bin centered received signal is a complex heterodyne applied to the received signal. Another option embeds the heterodyne in the filter weights of the channelizer. In this paper we showed that a channelizer with an odd number of paths and an odd number center frequencies in its IFFT algorithm had an interesting symmetry anomaly. The IFFT bin centers symmetric about DC include the DC bin but the bin centers symmetric about the half sample rate bracketed the half sample rate. We saw that the half sample rate resided midway between IFFT bins, the property we desired in the odd indexed channelizer. By translating DC to the half sample rate of a channelizer with an odd number of paths, we had the odd indexed channelizer without the complex heterodyne of data or filter weights. We then showed that under simple conditions, the sign reversals of the signal samples could be embedded in the polyphase filter weights so no operation was applied to input samples at the high input sample rate.

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